CSE 595 Independent Study

Graph Theory

Week 5

California State University - San Bernardino

Richard Vargas

Supervisor – Dr Owen Murphy

Chapter 3 Problem 45 (Spanning Trees)



Following Algorithm 3.22 in Chartrand [1],

Allow be the vertex set of Tree , where n is the length of the Prufer sequence, , given above. The edge set will update until the set .

Chapter 3 Problem 49 (Spanning Trees)



We may examine the graph

Therefore,

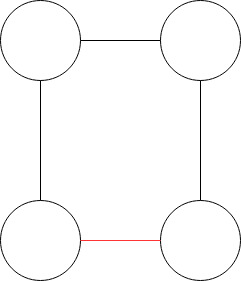
Since order in , then we have proven for an all paths of based on the properties of paths,

Chapter 3 Problem 59 (Spanning Trees)



By providing a counter example we can prove the above. Suppose that edge is bridge , designated as the red line, but does not belong to every spanning tree of graph . Chartrand [1] defines a bridge as,

*an edge in a connected graph whose removal results in a disconnected graph*

We may consider the following example graph.

The graph is a spanning tree that exists without , thus by contradiction it is shown that must exists in every spanning tree of a graph in order to be considered a bridge.

Chapter 3 Problem 63 (Spanning Trees)



When there is no graph that contains exactly two spanning trees, because there is only the graph that is a spanning tree.

For the case of it is obvious that only has one spanning tree, so this remains true.

For , looking at ,that has nodes and edges, spanning trees may be produce by removing one of the edges, always producing spanning trees.

Chapter 3 Problem 67 (The Minimum Spanning Tree Problem)



First following Algorithm 3.27 (Kruskal’s Algorithm) in Chartrand [1], an edge set will be built consisting of edges with the lowest values, so long as no cycles are induced.

Starting with the lowest edge weight, 3, the edges and , so adding on edge at a time,

Thus, a minimum spanning tree is induced by Kruskal’s Algorithm.

Next following Algorithm 3.29 (Prim’s Algorithm) in Chartrand [2], an edge set will be built, starting with any node, and adding the lowest weight adjacent edge and adding its adjacent node to the graph.

Starting with node , the smallest edge weight is 3 which connects to node ,

Next the smallest edge weight is 4, connecting to node either form node or node ,

Chapter 3 Problem 71 (The Minimum Spanning Tree Problem)



If the Kruskal’s Algorithm, it is obvious that the graph will have a distinct minimum spanning tree since

When adding any edge using this algorithm, there will always be a distinct weight, and therefore no alternative graphs will be induced.

Following Prim’s Algorithm, a distinct graph will also be induced. Given and node to start, selecting the minimum edge weight for each edge adjacent to the induced graph and adding it will cause for the same graph, if the graph is connected.

Works cited

“Trees.” *Graphs & Digraphs*, by Gary Chartrand et al., CRC Press, 2016, pp. 57–94.